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Let $s_1=[-r+\sqrt{(m^9+r^2)}]^{\frac{1}{3}}$, $s_2=[-r-\sqrt{(m^9+r^2)}]^{\frac{1}{3}}$, and the roots of (1) become $y_1=s_1+s_2$, $y_2=\frac{1}{2}(s_1+s_2)+\frac{1}{2}\sqrt{(-3)}(s_1-s_2)$, $y_3=-\frac{1}{2}(s_1+s_2)-\frac{1}{2}\sqrt{(-3)}(s_1-s_1)$.

$\therefore x^3+3mx=y_1$ or y_2 or y_3.........(2).

The nine roots of the three cubics (2) give the required solution. We let

$$s_3=\left(-\frac{y_1}{2}+\sqrt{(m^3+\tfrac{1}{4}y_1^2)}\right)^{\frac{1}{3}},\quad s_4=\left(-\frac{y_1}{2}-\sqrt{(m^3+\tfrac{1}{4}y_1^2)}\right)^{\frac{1}{3}}$$

and we denote the corresponding radicals when y_1 is replaced by y_2, by s_5, s_6; and when y_1 is replaced by y_3 by s_7, s_8. The nine roots of the given equation are:

$x_1=s_3+s_4$, $\qquad x_4=s_5+s_6$,

$x_2=-\frac{1}{2}(s_3+s_4)+\frac{1}{2}\sqrt{(-3)}(s_3-s_4)$, $\qquad x_5=-\frac{1}{2}(s_5+s_6)+\frac{1}{2}\sqrt{(-3)}(s_5-s_6)$,

$x_3=-\frac{1}{2}(s_3+s_4)-\frac{1}{2}\sqrt{(-3)}(s_1-s_4)$, $\qquad x_6=-\frac{1}{2}(s_5+s_6)-\frac{1}{2}\sqrt{(-3)}(s_5-s_6)$,

$x_7=s_7+s_8$,

$x_8=-\frac{1}{2}(s_7+s_8)+\frac{1}{2}\sqrt{(-3)}(s_7-s_8)$,

$x_9=-\frac{1}{2}(s_7+s_8)-\frac{1}{2}\sqrt{(-3)}(s_7-s_8)$.

Similarly solved by Elmer Schuyler, and J. Scheffer.

II. Solution by the PROPOSER.

Let $x=y+z$. Then $x^9+9mx^7+27m^2x^5+30m^3x^3+9m^4x+2r=0$, becomes

$$y^9+z^9+(yz+m)\,[9(y^7+z^7)+(36yz+27m)(y^5+z^5)+(84y^2z^2+105myz+30m^2)(y^3+z^3)+(126y^3z^3+189my^2z^2+81m^2yz+9m^3)(y+z)+2r=0.$$

$\therefore y^9+z^9=-2r$ and $yz=-m$.

$\therefore y^{18}+2ry^9=m^9$. $\quad\therefore y^9=-r\pm\sqrt{(r^2+m^9)}$.

Let $a^9=-r+\sqrt{(r^2+m^9)}$, $b^9=-r-\sqrt{(r^2+m^9)}$.

Let $w=\dfrac{-1+\sqrt{(-3)}}{2}=\beta^3$.

Then the nine roots are $a+b$, $\beta a+\beta^8b$, $\beta^2a+\beta^7b$, $\beta^3a+\beta^6b$, $\beta^4a+\beta^5b$, $\beta^5a+\beta^4b$, $\beta^6a+\beta^3b$, $\beta^7a+\beta^2b$, $\beta^8a+\beta b$.

Also solved by L. E. Newcomb, and F. D. Posey.

203. Proposed by L. E. NEWCOMB, Los Gatos, Cal.

The sum of a certain pair of roots of $x^4+ax^3+\left(\frac{2b}{a}+\frac{a^2}{4}\right)x^2+bx+c=0$ is equal to the sum of the remaining pair.

I. Solution by J. SCHEFFER, Hagerstown, Md.

The problem reduces to the problem of finding the value of m in the equation $a^4+ax^3+mx^2+bx+c=0$, subject to the condition that the sum of two roots equals the sum of the two others. Denoting the roots by α, β, γ, δ, we have $\alpha+\beta=\gamma+\delta=-\frac{1}{2}a$. Since $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=-b$, we have by factoring,

$$\alpha\beta(\gamma+\delta)+(\alpha+\beta)\gamma\delta=-b,\text{ or }(\alpha\beta+\gamma\delta)\tfrac{1}{2}a=b.$$

$\therefore \alpha\beta+\gamma\delta=2b/a$. Since $\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=m$, we have $(\alpha\beta+\gamma\delta)+(\alpha+\beta)(\gamma+\delta)=m$, or $(\alpha\beta+\gamma\delta)+(\alpha+\beta)^2=m$, or $2b/a+\frac{1}{4}a^2=m$

II. Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

Place $x^4+ax^3+(2b/a+a^2/4)x^2+bx+c=(x^2+kx+m)(x^2+kx+n)$ where k is the sum of two of the roots. Since the sum of two roots equals the sum of the other two, k must be the same in both factors. Equating like powers of x we get $2k=a$, $k^2+m+n=2b/a+a^2/4$, $k(m+n)=b$, $mn=c$.

$\therefore k=a/2$, $m+n=2b/a$, $mn=c$, etc. The roots are now easily found—

$$m=\frac{1}{a}[b\pm\sqrt{(b^2-a^2c)}],\quad n=\frac{1}{a}[\mp\sqrt{(b^2-a^2c)}].$$

$$\therefore x=-\tfrac{1}{2}[k\mp\sqrt{(k^2-4m)}]=-\tfrac{1}{2}\left(\frac{a}{2}\mp\sqrt{\frac{a^2}{4}-\frac{4}{b}[b\pm\sqrt{(b^2-a^2c)}]}\right).$$

$$x=-\tfrac{1}{2}[k\mp\sqrt{(k^2-4n)}]=-\tfrac{1}{2}\left(\frac{a}{2}\mp\sqrt{\frac{a^2}{4}-\frac{4}{a}[b\mp\sqrt{(b^2-a^2c)}]}\right).$$

Analogously solved by F. D. Posey.

III. Solution by A. H. HOLMES, Brunswick, Maine, and L. E. NEWCOMB, Los Gatos, California.

Transposing c and adding b^2/a^2 to both sides, the roots of the equation are easily found to be

$$x_1=-\tfrac{1}{2}\left[\frac{a}{2}+\sqrt{\frac{a^2}{4}-\frac{4}{a}[b+\sqrt{(b^2-4ac)}]}\right]$$

$$x_2=-\tfrac{1}{2}\left[\frac{a}{2}-\sqrt{\frac{a^2}{4}-\frac{4}{a}[b+\sqrt{(b^2-4ac)}]}\right]$$

$$x_3=-\tfrac{1}{2}\left[\frac{a}{2}+\sqrt{\frac{a^2}{4}-\frac{4}{a}[b-\sqrt{(b^2-4ac)}]}\right]$$

$$x_4=-\tfrac{1}{2}\left[\frac{a}{2}-\sqrt{\frac{a^2}{4}-\frac{4}{a}[b-\sqrt{(b^2-4ac)}]}\right]$$

Evidently, $x_1+x_2=x_3+x_4=-\frac{1}{2}a$.

Also solved by Elmer Schuyler.

204. Proposed by F. P. MATZ, Sc. D., Ph. D., Professor of Mathematics and Astronomy in Defiance College, Defiance, O.

If α, β, γ be the roots of the cubic equation $x^3+qx+r=0$, prove that $3\Sigma\alpha^2\Sigma\alpha^5=5\Sigma\alpha^3\Sigma\alpha^4$.